On Distributed Codes with Noisy Relays

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Abstract—**In this paper, we address the design of distributed coding schemes for the 3-node relay channel with half-duplex constraint. We first discuss the general problem of optimally re-encoding noisy data at the relay and summarize recently proposed approximations. Based on an analysis of the extrinsic information transfer characteristics of the component codes we investigate the potential of "weak" Turbo or LDPC codes which are employed by the source node in order to increase the reliability of the relay when decoding is not possible. The results show that we can indeed gain from the proposed code design.**

I. Introduction

Utilizing spatial diversity by employing multiple-antenna systems is a well-known concept to mitigate the effect of fading. However, for practical reasons, multiple antennas are not always applicable for a variety of mobile devices. An alternative concept is given by cooperative communications (see e.g. [1], [2]) where several nodes in a wireless network can achieve spatial diversity by relaying messages from each other.

Many different coding schemes were recently proposed in order to exploit cooperative diversity. Distributed channel coding schemes for the three-node relay channel like distributed Turbo codes (DTC) represent one important subset of these approaches. The traditional DTC setup (e.g. [3], [4]) represents a realization of the decode-and-forward protocol: the relay decodes the channel-encoded message which is broadcasted by the source node. If decoding is successful (e.g. determined by a CRC code), the relay performs re-encoding such that the overall code word received by the destination forms the code word of a Turbo code.

In order to mitigate the effect of error propagation at the relay and in order to make the DTC strategy applicable if the relay cannot be guaranteed to be error free, the soft decode-and-forward protocol was proposed in [5], [6]. Soft-input/soft-output (SISO) decoding algorithms are applied at the relay to realize "soft re-encoding". Based on the log-likelihood ratios (LLRs) for the information bits, which are provided by its channel decoder, the relay generates LLRs for the bits of the re-encoded code word. These LLRs are then transmitted either in an amplify-and-forward fashion as suggested in [5] or using the estimate-and-forward protocol [7] as proposed in [6]. Code design for such systems is addressed in [8].

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While the authors in [5], [6], [8] consider only the case of half-duplex relays with orthogonal channels between source and destination and relay and destination, [9] presents results for distributed coding schemes (based on convolutional and Turbo codes) in a block Markov coding framework for the non-orthogonal case and with a full-duplex relay. Related work based on low-density parity-check codes can be found, e.g., in [10]–[12].

In this paper we address the design of distributed coding schemes based on Turbo and convolutional codes for the special case where the relay is not able to decode correctly. We start with a summary of the optimal re-encoding functionality at the relay and of two of its approximations, the *soft* decode-and-forward strategy [5] and the *noisy* decode-and-forward strategy [13]. The second part of this paper focuses on code design aspects: for the channel code employed by the source node we analyze the tradeoff between the decoding threshold and the amount of information which is provided by the decoder when convergence cannot be achieved. Based on two examples we discuss then different coding strategies for the relay node. Our analysis is based on the extrinsic information transfer characteristics (EXIT charts, see e.g. [14]). As our results show we can gain indeed by employing "weaker" codes at the transmitter which are characterized by a higher decoding threshold but provide more information in situations where the relay cannot decode.

The remainder of the paper is organized as follows: Section II introduces the underlying transmission system and relaying protocol. Re-encoding strategies for the noisy relay channel are summarized in Section III. Sections IV and V discuss the code design and show the simulation results. The main results are summarized in the conclusions in Section VI.

II. Transmission System

We consider the transmission scheme depicted in Figure 1 which realizes a distributed parallel concatenated coding scheme with component codes $\mathcal{C}_S$ and $\mathcal{C}_R$ at the source and the relay, respectively.

The transmission is carried out in two phases: in a broadcast phase, a channel-encoded version $\boldsymbol{X}_S$ of the bit vector $\boldsymbol{B}$ with elements[1] $X_S \in \{-1, +1\}$ and $B \in \{0, 1\}$, respectively, is transmitted from the source node to the relay and the

[1] Indices are omitted for convenience, whenever this can be done without ambiguity.

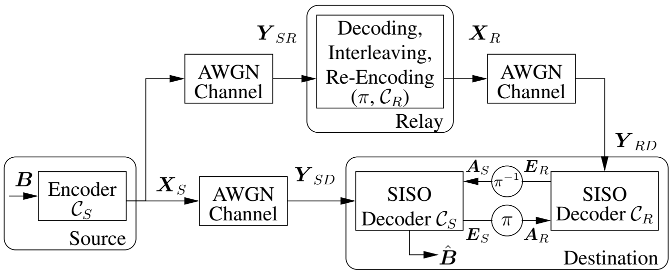

Fig. 1. Transmission model with distributed coding and iterative decoding.

destination. The relay decodes, interleaves, and re-encodes the received message considering the interleaver π and the code $\mathcal{C}_R$. During the second phase, the resulting symbol vector $\boldsymbol{X}_R$ is transmitted from the relay to the destination while the source keeps silent. An iterative decoder as shown in Figure 1 is employed by the receiver to decode the resulting distributed code. It is based on the component decoders for the codes $\mathcal{C}_S$ and $\mathcal{C}_R$. Throughout this paper the communication channels are realized by binary-input additive white Gaussian noise (BI-AWGN) channels parameterized either by the respective input-output mutual information $I(X;Y)$ or equivalently by the signal-to-noise ratio expressed in terms of E_s/N_0.

III. Re-encoding Strategies for Noisy Relays

In this section we consider the case where the relay cannot be guaranteed to be error free. We discuss the optimal decoding/re-encoding procedure at the relay as well as two approximations, the *soft decode-and-forward strategy* (sD&F) and the *noisy decode-and-forward strategy* (nD&F).

A. Optimal Soft and Hard Re-Encoding

Let in the following $X_{R_i}^{opt}$ denote the i-th code symbol of the relay's code word $\boldsymbol{X}_R^{opt}$ when the source-relay link is error free, i.e., $\boldsymbol{X}_R^{opt} = \mathcal{C}_R(\pi(\boldsymbol{B}))$.

In the general case where the source-relay link is not error free, the optimal re-encoding can be interpreted as a decoding problem where the relay provides *a posteriori* LLRs on the code symbols $X_{R_i}^{opt}$ based on the noisy observations in $\boldsymbol{Y}_{SR}$ and taking into account the interleaver π and the code $\mathcal{C}_R$ at the relay:

$$L(X_{R_i}^{opt}) = \log\left(\frac{\Pr(X_{R_i}^{opt} = +1|\boldsymbol{Y}_{SR})}{\Pr(X_{R_i}^{opt} = -1|\boldsymbol{Y}_{SR})}\right).$$

The derivation of the LLRs $L(X_{R_i}^{opt})$ requires knowledge of the *a posteriori* probability (APP) $\Pr(X_{R_i}^{opt} = x_i|\boldsymbol{Y}_{SR})$ which can be obtained by a marginalization of the APP $\Pr(\boldsymbol{B} = \boldsymbol{b}|\boldsymbol{Y}_{SR})$ over all hypotheses for the information word $\boldsymbol{b}$

$$\Pr(X_{R_i}^{opt} = x_i|\boldsymbol{Y}_{SR}) = \sum_{\boldsymbol{b}\text{ with } x_i \in \mathcal{C}_R(\pi(\boldsymbol{b}))} \Pr(\boldsymbol{B} = \boldsymbol{b}|\boldsymbol{Y}_{SR}),$$

taking again into account the interleaver π and the code $\mathcal{C}_R$. The relay can now forward the LLRs using an appropriate relaying function $f(.)$. The transmitted symbols X_R at the relay are then given by[2]

$$X_R = \beta \cdot f\left(L(X_R^{opt})\right)$$

with the normalization factor β such that the power constraint $\mathrm{E}\{X_R\} = P_R$ for the relay is matched. Table I gives a few examples for $f(.)$ for important relaying protocols.

Protocol	$f(x)$
Amplify and forward	x
Decode and forward	$\mathrm{sign}(x)$
Estimate and forward	$\tanh(x/2)$

TABLE I
Relaying protocols and relaying functions.

Remarks:

1) The re-encoding strategy discussed in this section is optimal in the sense that the LLRs $L(X_{R_i}^{opt})$ are a sufficient statistics for the optimal code symbols $X_{R_i}^{opt}$; i.e., no information is lost due to the re-encoding.
2) Clearly, the calculation of the APPs $\Pr(\boldsymbol{B} = \boldsymbol{b}|\boldsymbol{Y}_{SR})$ is not feasible; for practical implementations approximations are needed.
3) Employing decode and forward (D&F, see Table I) is optimal in the sense that the error probability $\Pr(X_R \neq X_R^{opt})$ at the relay is minimized. On the other hand it is known that forwarding the LLRs in an estimate-and-forward (E&F) fashion maximizes the signal-to-noise ration SNR_{RD} on the relay-destination link [7]. Clearly, both protocols need not to be necessarily the best choice when looking at the end-to-end quality of the entire coding scheme. In the context of distributed coding schemes, $f(.)$ should be chosen such that the mutual information between input and output of the relay-destination channel is maximized.
4) Iterative decoding at the destination can be carried out as illustrated in Figure 1 assuming that the component decoder associated with the relay is able to interpret the channel observations in $\boldsymbol{Y}_{RD}$ correctly as LLRs $L(Y_{RD}|X_R^{opt})$. This requires however additional *a priori* knowledge at the destination: in the case of E&F, a complicated non-linearity which relies on a precise model for the LLRs $L(X_R^{opt})$ has to be used to translate a received value y_{RD} into a LLR value $l = L(Y_{RD}\!=\!y_{RD}|X_R^{opt})$. Even the D&F strategy requires knowledge of the error probability $\Pr(X_R \neq X_R^{opt})$.

B. Soft Decode-and-Forward Protocol with Soft Re-Encoding

An efficient approximation of the optimal soft re-encoding strategy can be obtained as shown in Figure 2 and as suggested in [5], [6]. The same structure as in the conventional decode-and-forward protocol is used. However, instead of re-encoding estimates $\hat{B}$ on the source bits B LLRs $L(B)$ are passed through the interleaver to a soft-input/soft-output (SISO) re-encoder. It can be realized by another SISO decoder (e.g. [15]

[2]The index i is omitted in the following for convenience.

in the case where $\mathcal{C}_R$ is a convolutional code), and it provides estimates $\hat{L}(X_R^{opt})$ on the desired *a posteriori* LLRs $L(X_R^{opt})$.

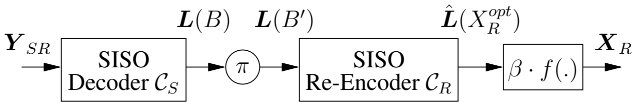

Fig. 2. Soft decoding and approximative soft re-encoding at the relay for the soft decode-and-forward strategy.

Again, the obtained soft values are mapped to the channel by considering an appropriate relaying function $f(.)$ (see e.g. Table I) and a normalization factor β accounting for the power constraint $\mathrm{E}\{X_R^2\} = P_R$ at the relay.

For decoding, the iterative decoder as shown in Figure 1 can be used in a straightforward manner taking however into account *Remark* 4).

C. Noisy Decode-and-Forward

In this section we propose a simple approximation of the optimal re-encoding procedure at the relay which was as well recently presented in [13]. Here, the relay employs the conventional D&F protocol as shown in Figure 3, however, ignoring the fact that the decoded bits at the relay $\hat{B}$ may not be equal to the originally transmitted source bits B.

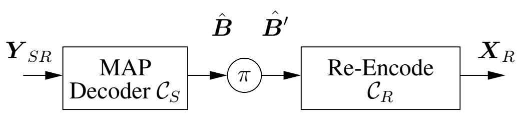

Fig. 3. Noisy decode-and-forward ignoring decoding errors at the relay.

While the encoding process remains the same in the noisy D&F case, the iterative decoder has to be modified. Since the information bits B and their estimates $\hat{B}$ are unequal with probability $\Pr(B \neq \hat{B}) = p$, the *a priori* information provided by the one decoder to the other is limited by $I(B; \hat{B}) = 1 - h(p)$. Accordingly, we have a decoding setup which is more similar to the joint decoding of correlated sources (as e.g. in the Slepian-Wolf problem), and the component decoders are coupled via the transition probabilities

$$\Pr(\hat{B}{=}\hat{b}|B{=}b) = \Pr(B{=}b|\hat{B}{=}\hat{b}) = \begin{cases} 1-p, & \text{for } b = \hat{b} \\ p, & \text{for } b \neq \hat{b} \end{cases} .$$

We can take this into account by extending the component decoder in Figure 1 which is associated with $\mathcal{C}_R$ as depicted in Figure 4. Here, the LLRs $L(\hat{B})$ for the noisy estimates $\hat{B}$ are converted into the LLRs $L(B)$ for the actual information bit B (and vice versa) by a *limiter* function $l_p(.)$, considering the transition probabilities $\Pr(\hat{B}{=}\hat{b}|B{=}b)$ and $\Pr(B{=}b|\hat{B}{=}\hat{b})$. For LLRs L_i and L_o and for $L_p := \log((1-p)/p)$, the input-output relation $L_o = l_p(L_i)$ of the limiter function $l_p(.)$ is given by

$$\begin{aligned} L_o &= \log\left(\frac{(1-p)\cdot\exp(+L_i/2) + p\cdot\exp(-L_i/2)}{p\cdot\exp(+L_i/2) + (1-p)\cdot\exp(-L_i/2)}\right) \\ &= \operatorname{sign}(L_i)\cdot\min(|L_i|, |L_p|) + \\ &\qquad \log\left(\frac{1+\exp(-|L_i+L_p|)}{1+\exp(-|L_i-L_p|)}\right). \end{aligned}$$

Note that $l_p(.)$ is a monotonically increasing function. Since the second term tends to zero whenever $|L_i| \gg |L_p|$, $l_p(.)$ limits the absolute value of the input LLRs to $|L_p|$.

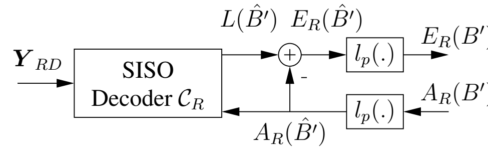

Fig. 4. Modified component decoder at the destination for the noisy decode-and-forward protocol.

IV. Code Design

In the following we limit our discussion on code design to Turbo codes and convolutional codes. Note however that an extension to low-density parity-check codes is straightforward.

A. Known Results

Distributed coding schemes over noisy relays have been previously analyzed in [8] for the case where the relay employs the sD&F strategy with soft re-encoding. The authors show that the channel code $\mathcal{C}_S$ at the source node should be realized by a Turbo code. The authors point out furthermore that re-encoding at the relay should be limited to non-recursive codes. This is due to the fact that in the case of recursive codes, the estimates on the LLRs $\hat{L}(X_R^{opt})$ will fade out with an increasing number of encoded bits (see [8] for details). The solution suggested in [8] is to use an uncoded transmission of the LLRs $L(B)$ with the estimate-and-forward protocol. One may hence interpret the resulting coding scheme rather as a relay-assisted Turbo code than as a distributed coding scheme. It is shown to outperform the design proposed in [5] which is based on a recursive convolutional code $\mathcal{C}_S$ at the source node and a non-recursive convolutional code $\mathcal{C}_R$ at the relay.

B. Design Tradeoffs for $\mathcal{C}_S$

In [16], the authors motivate that codes which show a good performance even if they cannot be correctly decoded, may lead to an improved code design for noisy relay channels. And in fact, a code design which 1.) maximizes the information available at the relay if convergence cannot be achieved, and 2.) leads to a low convergence threshold close to the theoretical limits at the same time would be desirable.

Inspired by this, our analysis focuses on two different choices for the channel code $\mathcal{C}_S$ at the source. Both codes are rate-$1/2$ codes which are implemented as Turbo codes. The first code $\mathcal{C}_S^{(High)}$ provides the relay with a relatively high amount of mutual information $I(B; L(B))$ when no additional *a priori* information is available and convergence cannot be achieved. The component codes are based on the rate-$1/2$ recursive systematic convolutional (RSC) code $(1, 13/15)_8$. The one component code is punctured to rate-$4/7$ by eliminating every 4-th parity bit, and the other component code is punctured to rate-4 by transmitting only every 4-th parity bit. The second code $\mathcal{C}_S^{(Low)}$ provides a low amount of information if decoding does not succeed. It is taken from [8],

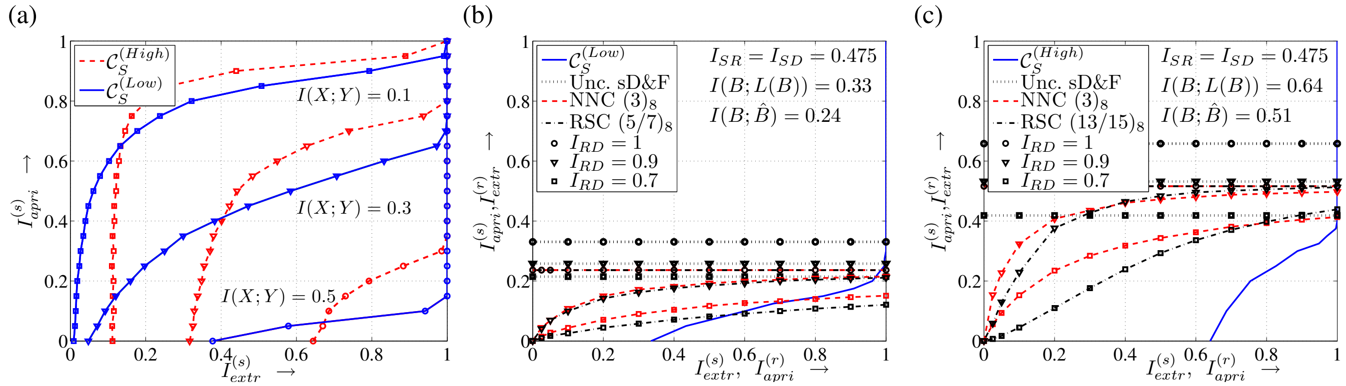

Fig. 5. EXIT function for the codes $\mathcal{C}_S^{(High)}$ (solid lines, (a)) and $\mathcal{C}_S^{(Low)}$ (dashed lines, (a)) transmitted over a BI-AWGN channel with $I(X;Y) \in \{0.1, 0.3, 0.5\}$ bit/channel use and EXIT chart analysis for different realizations of $\mathcal{C}_R$ in (b) and (c) for the noisy decode-and-forward strategy.

and it is formed by rate-1 component encoders with generator polynomials $(1/7)_8$ and $(5/7)_8$. The threshold in terms of E_s/N_0 for convergence without additional *a priori* knowledge for transmissions over a BI-AWGN channel is given by $T_{no\ apri}^{(Low)} \approx -2.17$ dB for $\mathcal{C}_S^{(Low)}$ and by $T_{no\ apri}^{(High)} \approx -1.54$ dB for $\mathcal{C}_S^{(High)}$. Clearly, $\mathcal{C}_S^{(Low)}$ is the better code for point-to-point communications.

The EXIT charts [14] for $\mathcal{C}_S^{(High)}$ and $\mathcal{C}_S^{(Low)}$ for a transmission over a BI-AWGN channel parameterized by its input-output mutual information $I(X;Y)$ are given in Figure 5(a). Note that these EXIT functions vary with the block length since the convergence behavior of the underlying iterative decoder relies on the block length. In Figure 5(a) we have $L = 10000$ information bits per block.

Let us now assume that the source uses $\mathcal{C}_S$ for its transmission. Assume furthermore that the source-destination link is parameterized by $I_{SD} = I(X_S;Y_{SD})$ and that we similarly have $I_{SR} = I(X_S;Y_{SR})$ for the source-relay channel. The mutual information $I(B;L(B))$ at the output of the SISO decoder at the relay is then given as $I(B;L(B)) = I_{extr}^{(s)}(0|I_{SR})$ since the relay has no additional *a priori* information available. On the other hand, we can define a convergence threshold $T_{\mathcal{C}_S}(I_{SD})$ for the decoder of $\mathcal{C}_S$ at the destination as

$$T_{\mathcal{C}_S}(I_{SD}) := \min \arg_i \left\{ I_{extr}^{(s)}(i|I_{SD}) = 1 \right\}. \tag{1}$$

It gives the amount of information which has to be at least provided by the decoder associated with the relay in order to achieve convergence for the overall code.

For a given rate pair I_{SR}, I_{SD} the code $\mathcal{C}_S$ has to be selected such that the relay gets enough information to make the code for the source-destination link converge, i.e., $I_{extr}^{(s)}(0|I_{SR}) > T_{\mathcal{C}_S}(I_{SD})$ has to be fulfilled. From Figure 5(a) we conclude however that $I_{extr}^{(s)}(0|I_{SR})$ and $T_{\mathcal{C}_S}(I_{SD})$ are coupled due to code properties; i.e., increasing the one will increase the other. Selecting a good code will hence mean to find a good tradeoff between $I_{extr}^{(s)}(0|I_{SR})$ and $T_{\mathcal{C}_S}(I_{SD})$.

Let us now focus on the case where the relay employs the nD&F strategy. One can show[3] that the mutual information $I(B,\hat{B})$ (see e.g. Figure 3) is given as

$$I(B,\hat{B}) = 1 - h_2\underbrace{\left(\frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{1}{2J^{-1}(I_{extr}^{(s)}(0|I_{SR}))}}\right)\right)}_{\doteq\, q(I_{extr}^{(s)}(0|I_{SR}))}. \tag{2}$$

Here, $h_2(.)$ denotes the binary entropy function, and $J^{-1}(I) = \sigma^2$ gives the relation between the input-output mutual information I of a BI-AWGN channel and its variance σ^2 [14]. With (1) and (2) we can now formulate a necessary condition which has to be fulfilled by $\mathcal{C}_S$ in order to enable convergence for a given rate pair I_{SR}, I_{SD}:

$$I(B,\hat{B}) = q(I_{extr}^{(s)}(0|I_{SR})) > T_{\mathcal{C}_S}(I_{SD}) \tag{3}$$

C. Matching $\mathcal{C}_R$

For simplicity we consider here the case where the channels between source and relay and source and destination have the same parameters $I_{SR} = I_{SD}$. $I_{extr}^{(s)}(0|I_{SR})$ and $T_{\mathcal{C}_S}(I_{SD})$ can then be obtained from the same EXIT function. For demonstration purpose we choose $I_{SR} = I_{SD} = 0.475$ bit per channel use. The resulting values for $I(B;L(B))$ and $I(B;\hat{B})$ are included for the two codes $\mathcal{C}_S^{(Low)}$ and $\mathcal{C}_S^{(High)}$ in the Figures 5(b) and (c), respectively.

The Figures 5(b) and (c) illustrate the behavior of different rate-1 codes $\mathcal{C}_R$ at the relay when the nD&F strategy is employed. As expected the maximum amount of extrinsic information is limited to $I(B;\hat{B})$. Accordingly, the typical behavior[4] of the recursive codes $(5/7)_8$ and $(13/15)_8$ cannot be observed any longer. For $I(B;\hat{B}) < 1$, the EXIT functions are more similar to those of non-recursive codes as a comparison to the non-recursive non-systematic convolutional (NNC) code $(3)_8$ shows. We can see furthermore that depending on the values of $I(B;\hat{B})$ and I_{RD} either the one or the other may be the better choice.

For comparison purpose, the EXIT functions for the uncoded sD&F strategy as suggested in [8] are included as well. Note that this implementation of sD&F is actually optimal in the sense that the generated soft output at the relay is a

[3] e.g. under a Gaussian assumption for the LLRs $L(B)$

[4] In contrast to non-recursive codes the EXIT charts of recursive codes achieve the point (1,1) for perfect decoding at the relay (i.e. for $B = \hat{B}$).

sufficient statistics for the transmitted information bits. The comparison visualizes as well the loss of information due to the hard-decision decoding at the relay.

Based on the EXIT chart analysis, one may now predict that a distributed coding scheme using the uncoded sD&F strategy will outperform the coded nD&F schemes. Furthermore, we can expect that schemes based on $\mathcal{C}_S^{(High)}$ will outperform those which are based on $\mathcal{C}_S^{(Low)}$.

V. Numerical Results

In order to verify the predicted behavior of the distributed codes from the previous section, we present numerical results obtained by Monte Carlo simulations. All channels are realized by BI-AWGN channels. The channel parameters E_s/N_0 for the source-relay and the source-destination channel are equal (i.e., $E_s/N_0^{(SR)} = E_s/N_0^{(SD)}$) and the parameter $E_s/N_0^{(RD)}$ for the relay-destination link is set to the fixed value $E_s/N_0^{(RD)} = 0$ dB. The block length is $L = 10000$ information bits per block. Figure 6 shows the bit error rate (BER) over the channel parameters $E_s/N_0^{(SR)} = E_s/N_0^{(SD)}$.

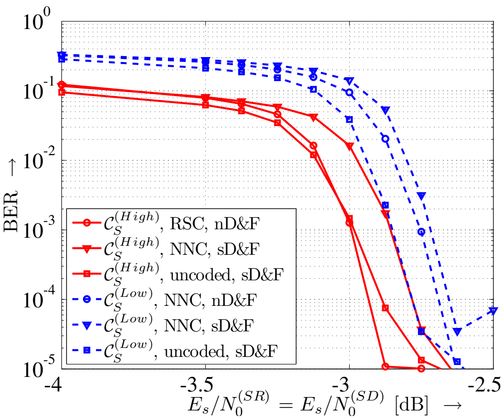

Fig. 6. Bit error rate (BER) versus the channel parameters $E_s/N_0^{(SR)} = E_s/N_0^{(SD)}$ for several combinations of $\mathcal{C}_S$ and $\mathcal{C}_R$ and $E_s/N_0^{(RD)} = 0$ dB.

Based on Figure 6 we can first conclude that the code $\mathcal{C}_S^{(High)}$ which is designed to increase the reliability of the relay in the case when it is not able to decode correctly, outperforms the code $\mathcal{C}_S^{(Low)}$, which provides a lower amount of information compared to $\mathcal{C}_S^{(High)}$. The best performance is achieved by the coded nD&F scheme using $\mathcal{C}_S^{(High)}$ and the recursive convolutional code $\mathcal{C}_R = (13/15)_8$ at the relay. A similarly good performance is shown by the uncoded sD&F scheme. However, the performance is worse than predicted in the previous section. We suspect that the performance loss is due to the mismatch of the LLRs $L(Y_{RD}|X_R)$ at the output of the relay-destination channel (see Remark 4) which prevents the decoder for $\mathcal{C}_S$ from correctly interpreting and fully exploiting the side information provided by the relay. A further loss can be observed for the channel coded sD&F schemes using the NNC code $(3)_8$. This loss is due to the soft re-encoding which reduces information. The loss in the mutual information $I(X_R; X_R^{opt})$ for the code $(3)_8$ can be quantified by the bounds on information combining provided by Theorem 3 in [17].

VI. Conclusions

In this paper, we have analyzed distributed coding strategies for the special case where the relay is not error free. We have provided a discussion on the optimal re-encoding functionality at the relay and its approximations. We have furthermore investigated the potential of a new code design where weak Turbo codes are employed by the source node in order to increase the reliability of the relay when error-free decoding cannot be obtained. Our results for the AWGN channel show that moderate gains can be obtained by this new strategy.

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